Concordia University

Department of Computer Science & Software Engineering

COMP 232/2 **Mathematics for Computer Science** Fall 2015

Assignment 1 Due date: October 1, 2015

1. For each of the following statements use a truth table to determine whether it is a tautology, a contradiction, or a contingency.

   (a) $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q)$

   (b) $((p \rightarrow r) \vee (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$

2. For each of the following logical equivalences state whether it is valid or invalid. If invalid then give a counterexample (*e.g.*, based on a truth assignment). If valid then give an algebraic proof using logical equivalences from Tables 6, 7, and 8 from Section 1.3 of textbook.

   (a) $p \rightarrow (q \rightarrow r) \;\equiv\; q \rightarrow (\neg p \vee r)$

   (b) $(p \rightarrow q) \vee (p \rightarrow r) \;\equiv\; (p \vee q) \rightarrow r))$

   (c) $((p \vee q) \wedge (\neg p \vee r)) \;\equiv\; (q \vee r)$

3. For each of the statements below, write it down in the form *"if p then q* , then write down the converse statement, and finally the contrapositive.

   (a) A positive integer is a prime only if it has no divisors other than 1 and itself.

   (b) To get an A in this class, it is necessary to do all the assignments.

   (c) Being born in Canada is a sufficient condition for Canadian citizenship.

   (d) You will reach the summit unless you begin your climb too late.

4. A set of propositions is *consistent* if there is an assignment of truth values to each of the variables in the propositions that makes each proposition true. Is the following set of propositions consistent?

   The system is in multi-user state if and only if it is operating normally.

   If the system is operating normally, the kernel is functioning.

   The kernel is not functioning or the system is in interrupt mode.

   If the system is not in multiuser state, then it is in interrupt mode. The system is in interrupt mode.

5. Let P and Q be predicates on the set S, where S has three elements, say, $S = \{a, b, c\}$. Then the statement $\forall x P(x)$ can also be written in full detail as $P(a) \wedge P(b) \wedge P(c)$. Rewrite each of the statements below in a similar fashion, using P, Q, and logical operators, but without using quantifiers.

   (a) $\forall x, y(P(x) \vee Q(y))$
   (b) $\exists x P(x) \wedge \exists x Q(x)$
   (c) $\exists x, y(P(x) \wedge Q(y))$
   (d) $\forall x \exists y(P(x) \wedge Q(y))$

6. Let $I(x)$ be the statement that x has an internet connection, and $C(x, y)$ be the statement that x and y have chatted with each other over the internet. The universe of discourse for the variables x and y is the set of all students in your class. Express each of the following using logical operations and quantifiers.

   (a) Not everyone in your class has an internet connection.
   (b) Everyone except one student in your class has an internet connection.
   (c) Everyone in your class with an internet connection has chatted over the internet with at least one other student in your class.
   (d) Someone in your class has an internet connection but has not chatted with anyone over the internet.

7. For each part in the previous question, form the negation of the statement so that all negation symbols occur immediately in front of predicates. For example:

$$\neg\Big(\forall x\big(P(x) \wedge Q(x)\big)\Big) \;\equiv\; \exists x\Big(\neg\big((P(x) \wedge Q(x)\big)\Big) \;\equiv\; \exists x\Big(\big(\neg P(x)\big) \vee \big(\neg Q(x)\big)\Big)$$

8. Determine the truth value of each of the following statements if the universe of discourse of each variable consists of all real numbers.

   (a) $\forall x \exists y(x + y = 1)$
   (b) $\exists x \exists y(x + 2y = 2 \wedge 2x + 4y = 5)$
   (c) $\forall x \exists y(x + y = 2 \wedge 2x - y = 1)$
   (d) $\forall x \forall y \exists z(z = (x + y)/2)$